1. Define linear transformation

2. Define kernel of a linear transformation.

3. What is the kernel of the linear transformation $L : P_3 \to P_2$ given by $L(p) = p'$. Here: P_3 is polynomials of degree 2 or less.

4. Find the transition matrix from the basis $\mathcal{B} = \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right)$ to the basis $\mathcal{B}' = \left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right)$. What column vector represents $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with respect to $\mathcal{B} = \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right)$. Use the transition matrix to find the column vector that represents $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with respect to $\mathcal{B}' = \left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right)$.

5. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} -2y \\ x+y \end{bmatrix}$$

What is the matrix representative of L with respect to the basis $\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right)$? Also, if v is represented by $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ with respect to $\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right)$ then what is the representative of $L\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix} \right)$ with respect to the same basis?

6. Use the transition matrix found earlier to find the matrix representative of L with respect to $\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right)$.

7. Define dimension.

8. Define nullity and rank for a both a matrix and a linear transformation.

9. Referring to problem 3, what is the matrix representative of L (differentiation) with respect to the bases $(1, x, x^2)$ and $(1, x)$ respectively.